

Appropriate best fit model for insurance claims modeling: A catalyst for managerial planning. Evidence from Ghana

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ABSTRACT: *It is always important to determine the distribution of insurance claims in order to estimate future expected values. This study seeks to determine the appropriate best fit model using sample data from Dormaa Health Insurance Directorate.*

Sample data collected from Dormaa Presbyterian Hospital of the Dormaa Municipal Health Insurance Scheme was analyzed using the Statistical Package for Social Science (SPSS), Excel spreadsheet, and Easy fit. It was found that the appropriate best fit model for the sample claims of the Dormaa Presbyterian Hospital follows lognormal distribution.

KEYWORDS: *Lognormal distribution, health insurance, national health insurance scheme*

A healthy population is major backbone for the socio-economic development of a nation. A healthy population with the requisite skills and knowledge become the backbone for economic development and transformation. It is in this light that every country in the world places premium on improved health care for her citizens. It has therefore become the state's responsibility to provide health care to the people of Ghana and this responsibility comes with financial challenges in view of the difficult economic issues that confront the country. Financing an efficient and effective health care system is of a major concern to countries all over the world, especially developing economies. In Ghana, healthcare financing has gone through several phases. After independence in 1957, the provision of health care in Ghana was financed by the state through tax revenue. However, it became obvious that this method of financing health care was not sustainable following the economic difficulties the country experienced from the beginning of the 1960s (Osei-Akoto, 2004). As part of efforts to revamp the Ghanaian economy, the state began to reduce expenditure on the provision of social services and the health sector witnessed considerable reduction in state funding. The then government in 1985, introduced user fees for all medical conditions, except certain specified communicable diseases. This system whereby people who access public health facilities pay user fees became known as 'cash and carry' and resulted in several operational challenges as well as people in the country (Asenso-Okyere, Osei-Akoto, Anum, & Appiah, 1997). In order to ameliorate the problems associated with the "cash and carry" system, government introduced the National Health Insurance Law, Act 650 in August, 2003. It sought to provide basic health care services to persons resident in Ghana through mutual and private health insurance schemes, and to establish a National Health Insurance Fund that will provide subsidy to licensed District Mutual Health Insurance

Schemes. Health Insurance is an alternative health care financing system which involves resource pooling and risk sharing among members (Ministry of Health, 2003). The Health Insurance Act mandates the creation of district-level Mutual Health Organizations (MHOs) in accordance with national guidelines and the establishment of a National Health Insurance Council (NHIC). The law represents a bold and innovative move by government to provide health insurance coverage to all of its citizens. This is meant to provide financial protection for the entire population and move away from the "Cash and Carry" system which was creating considerable equity concerns, largely due to the non-functional exemption mechanisms.

In view of the economic importance of National Health Insurance Scheme (NHIS) in developing countries, there is a need to use actuarial analysis to model the distribution of the claim amounts presented to the claim center, which can then be used to estimate the most appropriate credibility factor for the best expected claim for the near future.

LITERATURE REVIEW

Review of health insurance in Ghana

In order to mitigate the negative effects of the "Cash and Carry" system, especially on the poor, the government of Ghana commissioned various studies into alternative health care financing mechanisms. The study proposed that a centralized National Health Insurance Company should be set up to provide a compulsory "Mainstream Social Insurance Scheme". Also, the report recommended pilot "rural-based community financed schemes" for the non-formal sector but gave no further details as to how the Ministry of Health (MOH) was to do this (Aikens, 2005). In 1997, the NHIS pilot project was formally launched in the Eastern Region intended to cover four districts namely; New Juaben, Suhum/Krabo/Coaltar, Birim South and Kwahu South. As a result, a NHIS Secretariat was established to undertake the

preparatory work and carry out the NHIS programme. Soon after the implementation of the pilot scheme, there were a lot of debates about the strategic direction of health financing policy generally, and the pilot scheme in particular (Atim & Sock, 2000).

In 2003, the 'National Health Insurance Act' was passed to operationalize the policy decision to move from user fees towards a pre-payment financing mechanism. The Act explicitly requires every Ghanaian to join either a Mutual Health Insurance Scheme (MHIS) or a private mutual or commercial insurance scheme. To mobilize additional funds to support the implementation of the district MHIS, the government of Ghana instituted a National Insurance Levy of 2.5% on specific goods and services. This is intended to subsidize all fully paid contributions to the district health insurance schemes. In addition, 2.5% of social security contributions paid by formal sector employees. Those in the informal sector are expected to make direct contribution based on one's ability to pay. Children under 18 years whose parent(s) or guardian(s) pay their contributions and the elderly are exempted from paying any contribution. Diseases covered include malaria, diarrhea, upper respiratory tract infection, skin diseases, hypertension, diabetics, asthma, and a host of other diseases. The aim of health insurance is to spread the risks of incurring health care costs over a group of subscribers. Hence, the larger the subscribers the lower the risk burden on the individual and vice versa. Thus the design of the NHIS is based on the following principles; equity, risk equalization, cross-subsidization, quality care, efficiency in premium collection and claims administration, community or subscriber ownership, partnership and reinsurance (Ministry of Health, 2004).

Literature on actuarial modeling in insurance data

(Boucher, 2014; Hua, 2015; Wright, 2005) used actuarial modeling to fit models to many claim

amount drawn from consecutive years. He fitted analytic loss distribution using maximum likelihood estimation for each of the years. He used a lot of models such as Pareto, Burr, Inverse Burr, and Lognormal to fit his data and test to select the best fit one.

(Horg & Klugman, 1983) also used Weibull distribution to fit 35 observations of hurricane loss and they found that Weibull distribution performs as well as the lognormal distribution.

(Boucher, 2014; Renshaw, 2004) used actuarial modeling to fit a statistical distribution to some claim amounts, the statistical distribution he tested were Lognormal, Gamma, and Weibull distributions. He used maximum likelihood estimation to fit the distribution, an idea that was extended to his research, only that he based his methodology on the Bayesian solution. He then used the likelihood functions to calculate the posterior distribution.

(Cizek, Hardle, & Weron, 2005) explains that a typical models for insurance risk has two main component: one characterizing the frequency of claims (or incidence) of events, these are analyzed by discrete models such as poisson distribution, binomial distributions, and another describing the severity (size or amount) of loss resulting from the occurrence of the event, these are also analyzed by continuous models such as Normal, lognormal distribution, Gamma distribution, Weibull distribution etc.

METHODOLOGY

Basic concepts and definitions

In general insurance the severity and frequency of claims analysis is important because it helps in pricing and product development. The general insurance is based on the principle of pooling risk. Insurance involves a combination of risk pooling and risk transfer which reduces risk physically and monetarily (Baranoff, 2008). The continuous probability distributions considered are Normal, Log-normal, Pareto, Gamma, Weibull, and Exponential.

The normal distribution

Normal distribution is the type of continuous distribution that is most extensively used. The normal distribution is continuous and has 2 parameters, μ and σ ; they determine the location and scale, respectively (Casella & Berger, 2002). It is, therefore, described by its mean and variance. The mean of the distribution is referred to as the location parameter, and the (standard deviation) measures how the distribution is spread out (known as the scale parameter). Normal distribution is important in actuarial science and has a single peak at the center of the distribution. Generally, finance related random variables follow normal distribution, so knowledge about normal distribution is vital in understanding portfolio theory. The normal curve falls off smoothly in either direction from the central value which is the mean of the distribution. So the probability density function is bell-shaped and symmetrical about the mean. Its formula is given by:

$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Where $-\infty < x < \infty$ and $-\infty < \mu < \infty$ $\sigma > 0$

Its Mean and Variance are given as $E(X) = \mu$ and $VarX = \sigma^2$

The lognormal distribution

Log-normal distribution is normally used to determine the claim size distribution as it is positively skew; the mean of the data is greater than the mode and the random variable does not take negative values, which is a feature of claim size distribution. It is often used in modeling claim size (Boland, 2007; Czado, Kastenmeier, Brechmann, & Min, 2012). It has 2 parameters, mean μ which is the location parameter and standard deviation σ , the scale parameter (Hossack, Pollard, & Zehnwirth, 1983). Its probability density function is given by:

$$f(x, \mu, \sigma^2) = \frac{1}{\sigma x \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\log x - \mu}{\sigma} \right)^2 \right]$$

Where, $0 \leq x < \infty$, $-\infty < \mu < \infty$, $\sigma > 0$

The mean and variance are given by;

$E(X) = e^{\mu + \frac{\sigma^2}{2}}$ and $Var(X) = e^{2(\mu + \sigma^2)} - e^{2\mu + \sigma^2}$.
 x stands for loss or claim amount

The Pareto distribution

Pareto distribution is a positively skew, heavily –tailed distribution which is used to model claim size. It has two parameters, α , which is the shape parameter and β , the scale parameter. For the mean and variance to exist in Pareto distribution, α and β , must be greater than 1 and 2 respectively, (Cizek et al., 2005). Pareto is mostly used to model income distribution and insurance claims size. The probability density function of Pareto distribution is given by:

$$f(x/\alpha, \beta) = \frac{\beta \alpha^\beta}{(\alpha + x)^{\beta+1}}, x > 0, \alpha > 0$$

The mean and variance of Pareto distribution is given by;

$$E(X) = \frac{\alpha}{\beta-1}, \text{ where } \beta > 1 \text{ and } Var(X) = \frac{\beta \alpha^2}{(\beta-1)^2(\beta-2)}, \text{ where } \beta > 2$$

The Gamma distribution

Gamma distribution is used in the study of claim size and the analysis of heterogeneity of risk. It has two parameters, α which is the shape parameter and β the scale parameter.

The probability density function for gamma distribution is given by;

$$f(x/\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x},$$

where $x > 0, \alpha, \beta > 0$

$\Gamma(\alpha)$ is a number which depends on α . The Gamma distribution is not symmetrical; instead it is positively skew. The mean and variance are given by;

$$E(X) = \frac{\alpha}{\beta} \text{ and } Var(X) = \frac{\alpha}{\beta^2}.$$

The exponential distribution

The exponential distribution is another type of continuous distribution used in actuarial modeling of claims size in general insurance. The distribution has one parameter. The

probability density function for exponential distributions is given by;

$$f(x/\lambda) = \lambda e^{-\lambda x}, \text{ where } x > 0$$

The mean and variance of exponential are $E(X) = \frac{1}{\lambda}$ and $Var(X) = \frac{1}{\lambda^2}$

The Weibull distribution

The random variable x with Weibull distribution has a probability density function given by;

$$f(x/\alpha, \beta) = \alpha\beta x^{\beta-1} e^{-\alpha x^\beta}, \text{ where } x > 0, \alpha > 0, \beta > 0$$

It has two parameters α , which is the location parameter and β , the scale parameter. (Kleiber & Kotz, 2003) explain that the Weibull distribution has received maximum attention for the last ten years and is still growing strong, and (Horg & Klugman, 1983) noticed its potential in insurance loss.

Its mean and variance are given by;

$$E(X) = \alpha^{-\frac{1}{\beta}} \Gamma\left(1 + \frac{1}{\beta}\right), Var(X) = \alpha^{-\frac{2}{\beta}} \left[\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right) \right]$$

Research design

Secondary data was used from Dormaa Municipal Health Insurance Scheme, regarding their claims presented from the health centers (January 2008-December 2014). Two assumptions were made on the data:

1. All the claims came from the same distribution (they are independent and identically distributed). This means that there is no correlation between reported claims.
2. All future claims were to be generated from the same distribution, that is, the future claims are to follow the same distributions as the past claims.

Population and sample size

The targeted population for the study was made up of all the 28 accredited health care providers in Dormaa Municipal that submit claims to the

Municipal Health Insurance Scheme. However, the sample size used for the study was Dormaa Presbyterian Hospital because their claim amounts are sent directly to the head office of national health insurance Scheme in Accra

Sample size determination and sampling

The sample size that was used to determine the likelihood function of claim amount was based on the formula below:

$$n = \frac{Z_{\alpha}^2 \sigma^2}{\epsilon^2}$$

Where, ϵ = how close should the sample estimates be to the unknown parameter (bound)

Z = the confidence level

σ = the estimates of the standard deviation of the population.

We needed the sample estimate to be 0.065% away from the unknown parameter, with 95% confidence level. The sample was drawn from a population with a random mean of 393120 and variance of 632970 as shown in table 4.1. The ϵ margin was determined as;

$$\epsilon = \frac{0.065}{100} \times 393120 = 258 \text{ (bound).}$$

We will like to be able to determine the sample estimate, the mean or average claim amount of Dormaa Municipal Health Insurance Scheme to be within ± 258 cedis, with 95% ($Z = 1.96$) confidence level. The sample size was then determined as;

$$n = \frac{1.96^2 \times 632970}{(258)^2} = 36$$

Simple random sampling

Simple random sampling techniques were employed to select sample of claim amount from Dormaa Presbyterian Hospital given that its parameter is the same as the unknown population parameter, for determining the likelihood function. 36 monthly aggregate claim amounts were selected from the secondary data provided by the hospital to form the sample size, such as;

$$(x_1, x_2, x_3, \dots, \dots, \dots, \dots, \dots, \dots, x_n)$$

Chi-square goodness of fit tests

The chi-square goodness-of-fit test is used to test how well the distribution fits a given data set. (Boland, 2007) explains that when testing the fit of a continuous distribution, the data is usually first binned (or grouped) into k intervals. We then calculate the number of expected observations E_i based on a grouped data and compare it with the actual observed numbers O_i for each interval.

Where:

$$E_i = \int_a^b (\text{probability density of sample}) dx \times \text{samplesize}$$

$$E_i = \int_a^b f(x) \times \text{samplesize}.$$

We then measure the fit of the distribution which is obtained from the test statistic:

$$X^2 = \frac{(O_i - E_i)^2}{E_i}$$

If the value of the calculated chi-square test statistic is large, we will reject the distribution being considered since it signifies a lack of fit between the observed and expected values (Montgomery & Runger, 2003).

RESULTS

Descriptive statistics of the population data

The summary statistics of the aggregate monthly claim amount was used in pointing out the salient features of the data in order to determine the distribution for the population data. The descriptive statistics obtained are the mean, mode, median, skewness, etc. These are obtained by using SPSS and Excel spreadsheets on the 84 data points from the scheme office. The logic behind the best model selection criteria will be based on the results of summary statistics and the test statistics.

Descriptive statistics of the sample data

The descriptive statistics of the aggregate monthly claim amount was used in pointing out some features of the data in order to determine the best distribution for the sample data. The

descriptive statistics obtained are the mean, mode, median, skewness, kurtosis etc. These are obtained by using SPSS on the 36 data points from the Dormaa Presbyterian Hospital. The idea behind the descriptive statistics results is to identify distribution family that the aggregate claim from the sample will follow. The summary statistics is presented in table 4.2.

The descriptive statistics of the sample data shows that the mode (15.310) < mean (21.2090). This shows that the distribution families that can be used to model the sample claims amount from the Dormaa Presbyterian Hospital are Exponential, Log-normal, Pareto Weibull, Gamma etc. The main distributions that were considered under this study to model the sample claims are the following four: Exponential, Log-normal, Gamma and Weibull.

Parameter estimation

Given any model, there exists a great deal of theories for making estimates of the model parameters based on the empirical data. In this case the sample claim data was used to compute the parameters of the selected distributions. Whenever the parameters of any distribution are obtained using sample data, it implies that, the statistical distribution has been fitted to the claims data. The parameters were computed by using Easy fit and SPSS. In this regard table 4.3 below shows the parameters of the distributions. Where α was taken to be the value of the first parameter (shape parameter) and β was taken to be the second parameter (Scale parameter)

The parameters obtained were used to derive the probability density function (pdf) and the expectation of each distribution for the sample data. The probability density function of each distribution is then used in the computation of probabilities from bin or group ranges in the sample data.

Determining probability density function (pdf) and expectation

The probability density functions of the above distributions are paramount in estimating

probabilities of each distribution at a given interval. The pdf and expectation of the distribution are determined based on the parameter values calculated in table 4.3. The 36 sample aggregate monthly claim amounts from Presbyterian hospital are random variable which can assume values such as, $x_1, x_2, x_3, \dots, x_{36}$. The pdf for each distribution were determined to help the researchers to calculate the probability of each distribution after the sample data have been binned or grouped. The probabilities obtained are used to calculate the expected claims in group interval to compare to actual claims. The probability density functions (pdf) are shown in table 4.4. The pdf of the models were determined for calculating probabilities of all the distributions at the given intervals in table 4.4.

Determining probabilities of distributions

The sample aggregate claims from the Municipal hospital were grouped or binned to six classes to know the actual claims for each interval. The probabilities for each interval were then calculated for every individual distribution considered in the study. The probabilities were computed using cumulative distribution function of the distributions and excel spreadsheet.

The probability of each range is essential in determining the expected claims for each distribution. Table 4.5 shows all the actual aggregate claims for each interval and their corresponding probabilities after the sample data have been binned or grouped

Determining expected aggregate claims

In other to be able to determine the best fit distribution for the aggregate claims from the municipal hospital, the expected claims for each bin or group intervals were estimated from each distribution. This enables the application of chi-square for determining the best fit model for the aggregate claim, using the minimum chi-square value. The sample size was multiplied by the

probabilities of each interval to estimate the expected claims for the distributions using the formula below. Table 4.6 shows the expected claims of the distributions,

$$E_i = \int_a^b f(x)dx \times \text{samplesize}$$

Determining Chi-square values

The chi-square goodness-of-fit test is used to test how well the distribution fits a given data set. We calculate the number of expected observations E_i based on a grouped data and compare it with the actual observed numbers O_i for each interval. We then measure the fit of the distribution which is obtained from the test statistic;

$$\chi^2 = \frac{(O_i - E_i)^2}{E_i}$$

If the value of the chi-square test statistic is large, we will reject the distribution being considered since it signifies a lack of fit between the observed and expected values. Table 4.7 shows the chi-square values computed from table 4.6. In addition, goodness of fit test and p-p plot will be run to ensure that we draw the most accurate conclusion.

DISCUSSIONS

Comparing the chi-square values of the four distributions for the best fit model, the minimum chi-square value is 1.111 corresponding to lognormal distribution. Also the Kolmogorov Smirnov and Anderson Darling test do not reject Log-normal as a good fit for the sample data at 5% significant level. The result of the test is shown in table 4.8 above. In addition, the probability plot (P-P) in figure 4.1 above also shows clearly that there is a close fit of the data pointing to log-normal distribution.

Therefore the approximate best fit model for the sample data is Log-normal distribution. We then conclude that Log-normal distribution signifies the best fit model out of the four distributions for Dormaa Presbyterian hospital sample monthly aggregate claims.

CONCLUSION

The lognormal distribution is commonly used to model the lives of units whose modes are a fatigue-stress in nature and, it can have widespread application. It is a good companion to the Weibull distribution when attempting to model these types of units. It is used extensively in reliability applications to model failure times. It starts at zero and runs to positive infinite, thus is skewed right and often useful in modeling claim size (Horg & Klugman, 1983). As the probability function (pdf) suggests, the lognormal distribution is the distribution of a random variable x in a log space. If the data size is too large, then in most cases it is assumed to be approaching normal distribution. Depending on the value of the standard deviation the distribution may appear similar to exponential distribution or the normal distribution.

In the insurance area, especially, based on observations of claims corresponding to an exposure, and on observations of the total amount of claims incurred, the risk theory arises to quantify risks and to fit company ruin. However, the main problem is the complexity to obtain the distribution function and consequently, the likelihood function used to calculate the estimation of the parameters. Choosing the most suitable best fit model and claim distribution will help Actuaries to use Bayesian's credibility theory to estimate the most appropriate credibility factor for the best expected claim for the near future.

Claim modeling is very important, since a good understanding and interpretation of claim distribution is the back-bone of all the decisions made in the insurance industry regarding expected profits and reserves necessary to ensure profitability and the impact of re-insurance (Boland, 2006). This study, therefore, seeks to provide further empirical details for the understanding of the best fit model and distribution of claims amount submitted to National Health Insurance Scheme offices.

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APPENDIX

Table 4.1 Summary Statistics of population (× 0000)

N	Mean	Median	Mode	Std deviation	Variance	Skewness	Range	Sum
84	39.312	38.469	36.456	7.9557	63.297	0.028	32.414	3302.2

Source: Dormaa Municipal Health Insurance, computed by Authors

The 95% confidence level for the mean is $37.586 \leq \mu \leq 41.039$.

Table 4.2 Descriptive statistics(× 0000)

N	Mean	Median	Mode	Std deviation	Variance	Skewness	Range	Sum
36	21.2090	20.3900	15.3100	2.9328	8.601	0.121	11.549	763.530

Source: Dormaa Presbyterian Hospital, computed by Authors

The 95% confidence level for the mean is $20.0300 \leq \mu \leq 21.9430$.

Table 4.3 Parameters estimates

Model	α	β
Exponential	0.04715	
Log-normal	3.0451	0.137
Gamma	52.297	0.4055
Weibull	8.215	22.283

Source: Dormaa Presbyterian Hospital, computed by the Authors

Table 4.4 Probability density function and expectation estimates

Model	PDF	Expectation

Exponential	$\alpha e^{-\alpha x}, x > 0$	21.209
Log-normal	$\frac{1}{\sigma\sqrt{2\pi}} \frac{1}{x} e^{\left\{-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2\right\}}$ $x > 0, \mu > 0, \sigma > 0$	22.502
Gamma	$\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, x, \alpha, \beta > 0$	128.969
Weibull	$\alpha\beta x^{\beta-1} e^{-\alpha x^\beta}, x > 0$	0.9299

Source: Dormaa Presbyterian Hospital, computed by Authors

Table 4.5 Probability estimates

Amount	Actual (frequency)	Exponential	Log- normal	Gamma	Weibull
< 17	2	0.5502	0.06097	.06733	0.1026
17 – 19	8	0.0404	0.17037	0.1640	0.1340
19 – 21	10	0.0367	0.2670	0.2585	0.2224
21 – 23	7	0.0390	0.2469	0.2491	0.2676
23 – 25	5	0.0304	0.15227	0.1585	0.1970
25 <	4	0.3080	0.1024	0.1024	0.0762
Total	36	1	1	1	1

Source: Dormaa Presbyterian Hospital, computed by researchers

Table 4.6 Expected claim and actual claims

Amount	Actual O_i	Exponential E_i	Lognormal E_i	Gamma E_i	Weibull E_i
< 17	2	20	2	2	4

17 – 19	8	2	6	6	5
19 – 21	10	1	10	9	8
21 – 23	7	1	9	9	9
23 – 25	5	1	5	6	7
25 <	4	11	4	4	3
Total	36	36	36	36	36

Source: Dormaa Presbyterian Hospital, computed by the researchers

Table 4.7 Chi-square values of distributions

Model	Chi-square values
Exponential distribution	171.655
Log-normal distribution	1.111
Gamma distribution	1.333
Weibull distribution	4.640

Source: Dormaa Presbyterian Hospital, computed by the researchers

Table 4.8 Goodness of fit test

Lognormal					
Kolmogorov-Smirnov					
Sample Size	36				
Statistic	0.16015				
P-Value	0.28297				
Significant level	0.2	0.1	0.05	0.02	0.01

Critical Value	0.17418	0.1991	0.22119	0.24732	0.26532
Reject?	No	No	No	No	No
Anderson-Darling					
Sample Size	36				
Statistic	0.83146				
Significant level	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	No	No	No	No	No

Source: Dormaa Presbyterian Hospital

Figure 4.1 lognormal P-Plot of Dormaa Presbyterian Hospital Data

